NEWPORT GIRLS' HIGH SCHOOL ACADEMY TRUST

A level Further Maths

Summer Work

Introduction

Congratulations on choosing to study A Level Further Maths. To help you prepare, this booklet will enable you to brush up on some of the skills you have learned at GCSE. You are going to need to use them from day 1, and if you don't have a good grasp of the basics you need to work on them NOW so that you can start with confidence.

- **You can do your work in a folder or exercise book**
- **You will need to present your work neatly and show all your working.**
- **The method is more important than the answer.**
- **You need to mark you work, clearly showing correction in green pen**
- **If you get something wrong, revise the topic then try again. The aim is to get EVERYTHING right!**
- **You will need to redo any questions you find challenging so that you are fluent with all the skills.**
- **You will need to bring your completed work to your first lesson.**

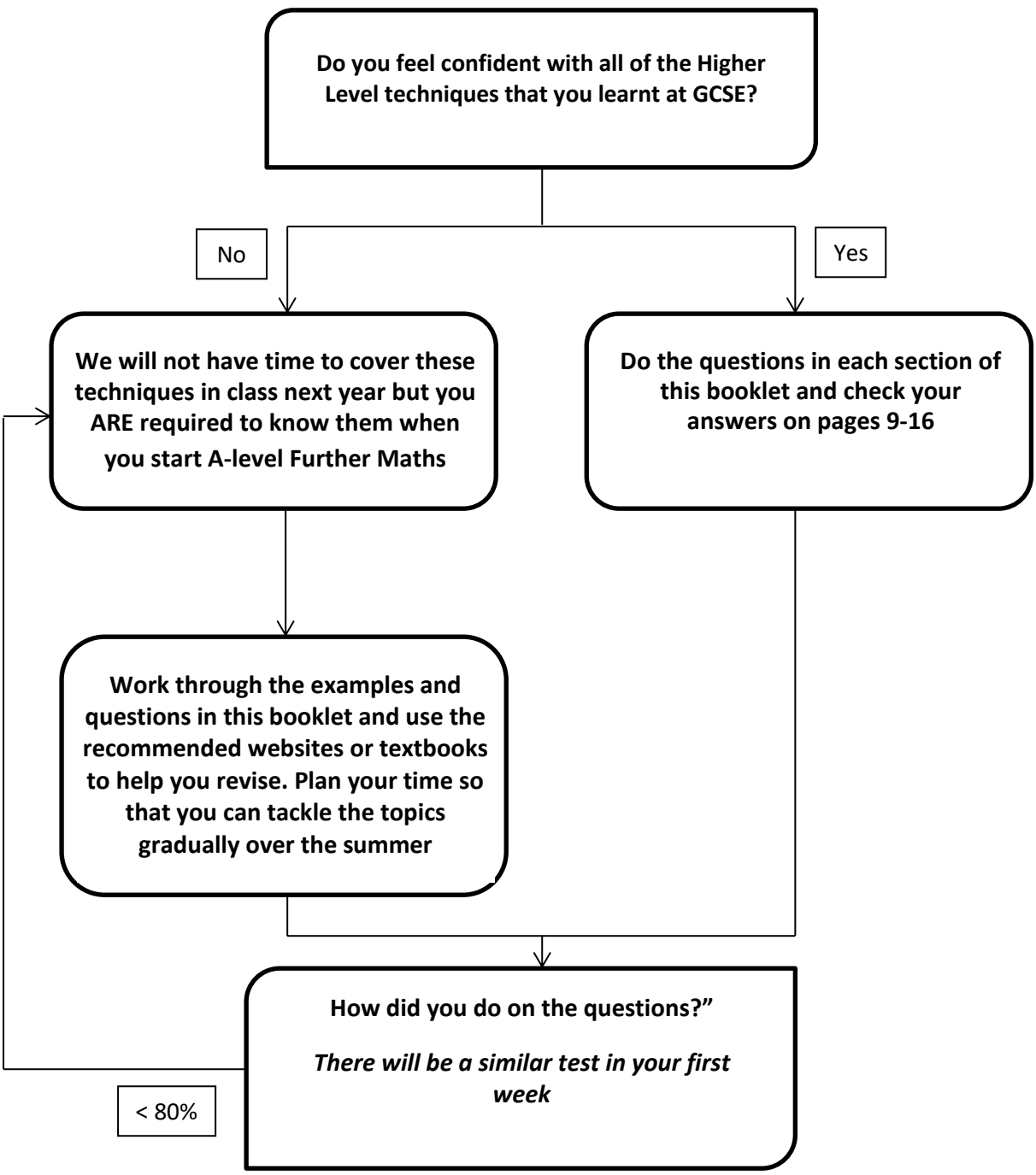
Studying A-Level Further Maths is about learning how to solve problems, and getting stuck is part of the learning process. You should expect to get stuck while working through this booklet but these are all GCSE techniques that you will need to master.

There are loads of great resources on the internet to help you, including ExamSolutions and MathsGenie which contains video tutorials for all GCSE Higher content. We also recommend using these sites throughout the A Level course as well as TL Maths (https://sites.google.com/view/tlmaths/home), PhysicsandMathsTutor, A-levelMathsRevision and SaveMyExams

Expectations of an A-Level Further Mathematics Student:

- For each hour you have in class, you are expected to complete one hour of work outside of lessons.
- You will need to keep neat, accurate and well-ordered notes. You will be supported by your teachers to achieve this but the majority of effort will come from you.
- It's the method that matters, not the answer. Often you are given the answer but need to show steps in the method. How you present your work can make a big difference to whether you get the right answer at all and whether anyone can understand your method.
- It is okay to make mistakes and feel stuck when completing problems. A good student will come and seek support from their teachers to develop their understanding of these more difficult topics.

We will not have time to cover these techniques in class next year but you ARE required to know them when you start A-level Further Maths.



Number

1) Which of the following are integers?

3 -2.8 0.4 $\frac{3}{4}$ 7.92 -9 202 0

2) Which of the following values are rational and which are irrational?

$4.\dot{7}$ π $\sqrt{8}$ $\frac{1}{5}$ -7 $\sqrt{16}$ 12.452 3.1

3) If $0 < x < 1$, compare the size of $x - \frac{1}{x}$ to $x^2 - x$

Indices, expanding and factorising

4) If $2^{2x+1} \times 4^{x+1} = 8^{x+2}$, find the value of x

5) Factorise the following:

$5y(y-1) + 3(y-1)$ $pq^2 - p^2q$ $16m^2 - 81n^2$

6) Multiply out the brackets and simplify where possible:
$(x-2)(2x+3)(x+7)$ $-(x-3)(x-1)(3x+2)$ $(x+1)(x-1)(x+5)(4x-1)$

7) $(x-3)(2x+1)(Ax+1) \equiv 8x^3 + Bx^2 + Cx - 3$ Work out the value of A, the value of B and the value of C.

Inequalities

8) Solve the following:

$8x + 3 \leq 4x$ $3(4-x) > 3$ $3x^2 + 2 < 14$

$7x^2 - 4 \geq 59$ $x^2 - 4x + 10 \geq 2x + 5$

9). Draw a set of axes, show the region that satisfies the following inequalities:

$y > 3x - 2$ $y < x + 2$ $y + x > -1$

Functions and Proof

10) $f(x) = \frac{x+5}{3}$ and $g(x) = x - 3$

Evaluate $f(4)$ Find $fg(x)$ Find $f^{-1}(x)$

11). $f(x) = 3x^3 - 2x^2 + 4$ Express $f(x+2)$ in the form $ax^3 + bx^2 + cx + d$

12) a) Express $x^2 + 6x + 11$ in the form $(x+a)^2 + b$ where a and b are integers
b) Hence, prove that $x^2 + 6x + 11$ is always positive

Drawing graphs and transformations of curves

13) A curve has the equation $y = 2x^2 - 5x + 12$

a) Write the curve in the for y = $a\,(x+m)^2 + n$ and hence find the minimum points of the graph.

b) Does the graph cross the x-axis? If yes, find the coordinates of the point of intersection.

14). On separate axes, sketch the following graphs:

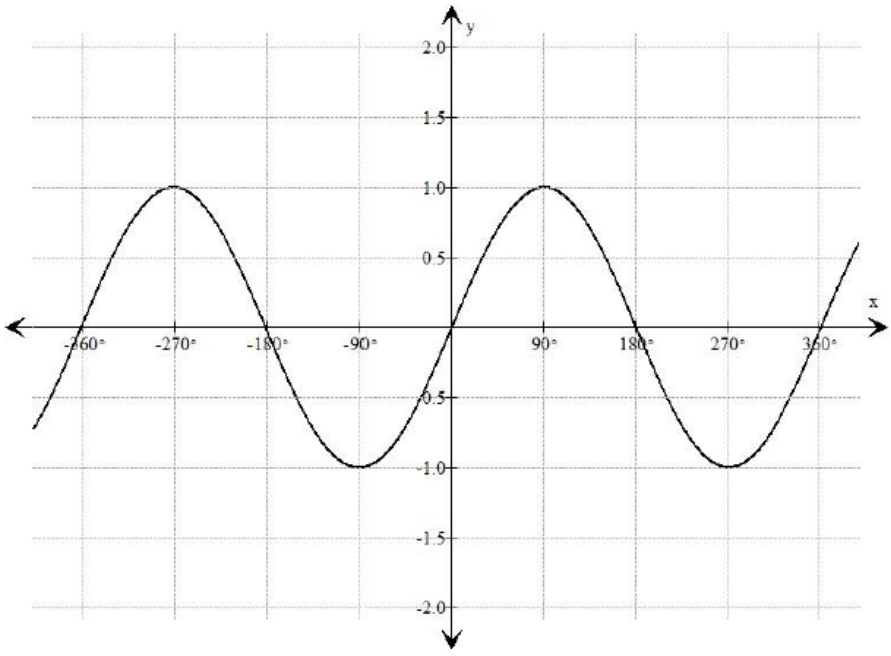
a) $y = -x^3$ $b)$ $y = \frac{-3}{x}$ c) $y = \frac{1}{x} + 1$ d) $y = \frac{2}{x^2}$

15) The graph of $y = \sin(x)$ is plotted below. Sketch the following transformations of $y = \sin(x)$on the same set of axes:

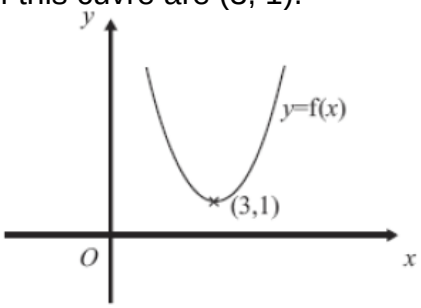
a) $y = 2\sin(x)$

b) $y = \sin(4x)$

c) $y = \sin(x - 90)$



16) The diagram shows part of the curve with equation $y = f(x)$. The coordinates of the minimum point of this cuvre are (3, 1).



Write down the coordinates of the minimum point of the curve with equation:

a) $y = f(x) + 3$

b) $y = f(x - 2)$

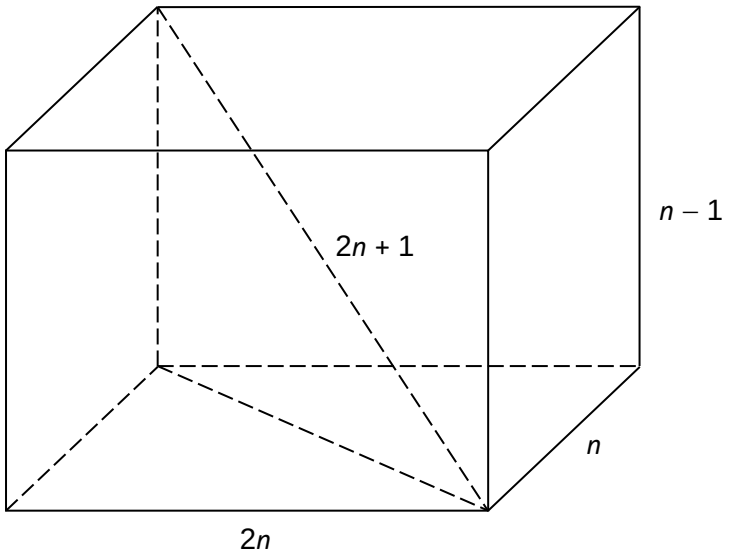
c) $y = f\left(\frac{1}{2}x\right)$

3D Trigonometry and Pythagoras' Theorem

17) A cuboid has dimensions $2n$, n and $n - 1$ cm.

A diagonal has length $2n + 1$ cm.

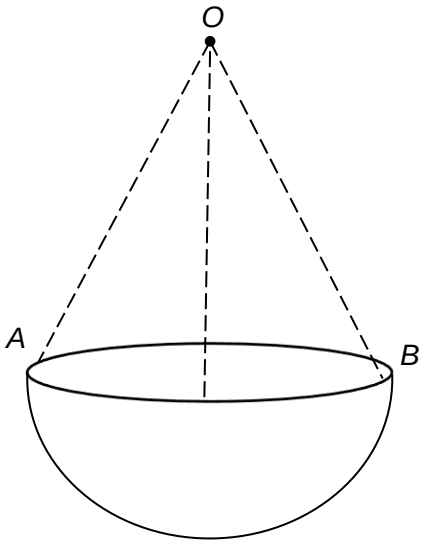
Not drawn accurately



Work out n.

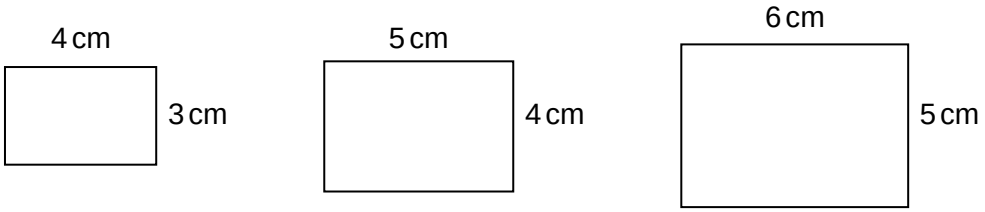
18) A hanging basket is made from a hemisphere and three chains.
The radius of the hemisphere is 10 cm.
Each chain is 30 cm long.
The chains are equally spaced around the rim of the hemisphere.

Work out angle *AOB*.



Sequences

19)



This pattern of rectangles continues.

Show that the sequence of numbers formed by the areas of these rectangles has *n*th term

$$n^2 + 5n + 6$$

20)

A linear sequence starts

$$a + b \qquad a + 3b \qquad a + 5b \qquad a + 7b \qquad \ldots\ldots\ldots\ldots$$

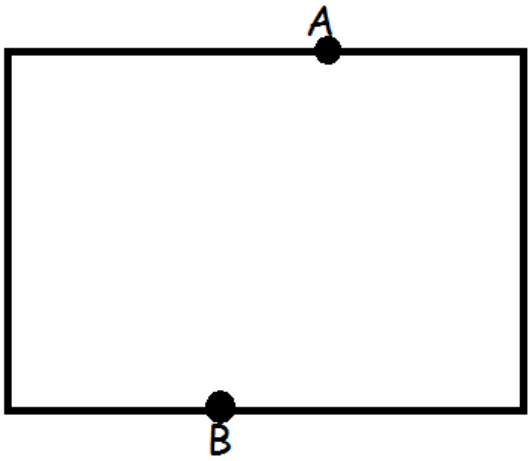
The 5th and 8th terms have values 35 and 59.

(a) Work out *a* and *b*.

(b) Work out the *n*th term of the sequence.

Transformations and Loci

21)



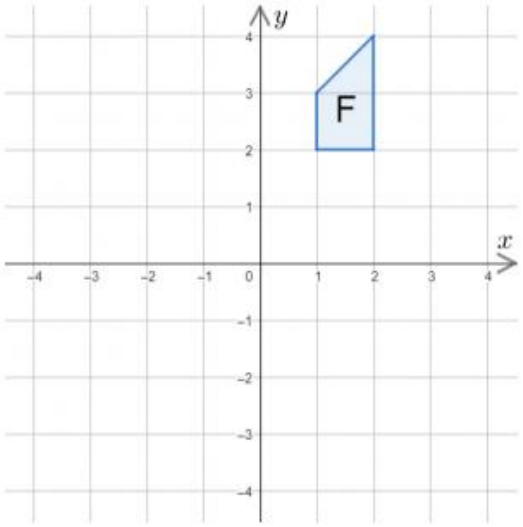
A snail moves so that it is always within the rectangle and is equidistant from points A and B. Use ruler and compasses to show where the snail moves.

22) In this order, perform the following two transformations to shape F.

a) Rotation $180°$ clockwise about (1,2)

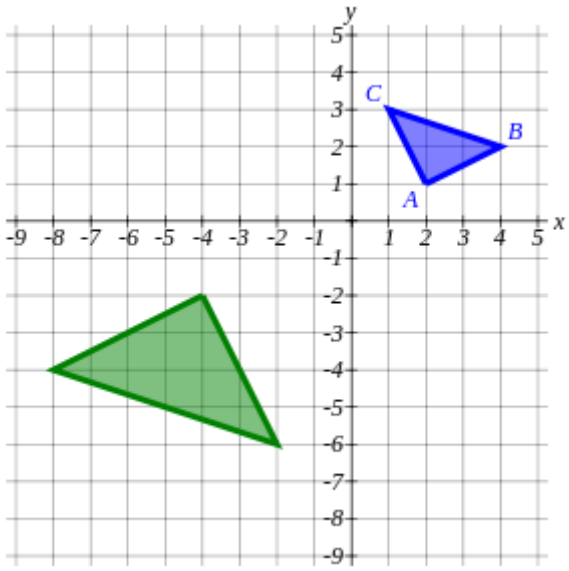
b) Reflection in the line $y = x$

Mark the resulting shape with a G. Extension: Are there any invariant points?



23)

Fully describe the single transformation from the triangle ABC to its image.



Vectors

24)

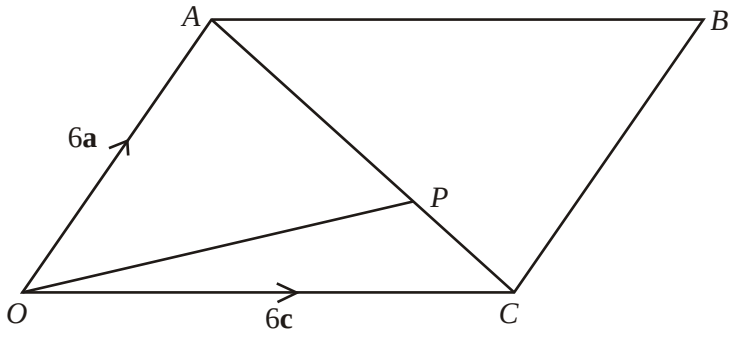


Diagram **NOT** accurately drawn

OABC is a parallelogram.

P is the point on *AC* such that $AP = \frac{2}{3}AC$.

$\overrightarrow{OA} = 6\mathbf{a}$. $\overrightarrow{OC} = 6\mathbf{c}$.

(a) Find the vector $\overrightarrow{OP}$.
Give your answer in terms of **a** and **c**.

The midpoint of *CB* is *M*.

(b) Prove that *OPM* is a straight line.

Answers

Number

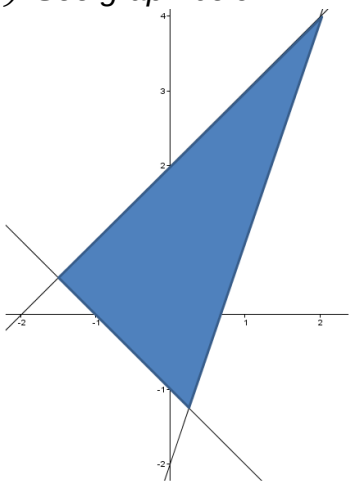
1) 3, -9, 202, 0
2) Rational: 4.7, 1/5, -7 $\sqrt{16}$, 12.451, 3.1
   Irrational: $\sqrt{8}$, π
3) $\left(x-\frac{1}{x}\right)>\left(x^2-x\right)$

Indices, expanding and factorising

4) $x = 3$
5) $(y-1)(5y+3)$ $pq(q-p)$ $(4m+9n)(4m-9n)$
6) $2x^3+13x^2-13x-42$ $-3x^3+10x^2-x-6$ $4x^4+19x^3-9x^2-19x+5$
7) $A=4$ $B=-18$ $C=-17$

Inequalities

8) $x \leq -\frac{3}{4}$ $x<3$ $-2<x<2$ $x \geq 3$ *or* $x \leq -3$ $x \leq 1$ *or* $x \geq 5$
9) See graph below



Functions and Proof

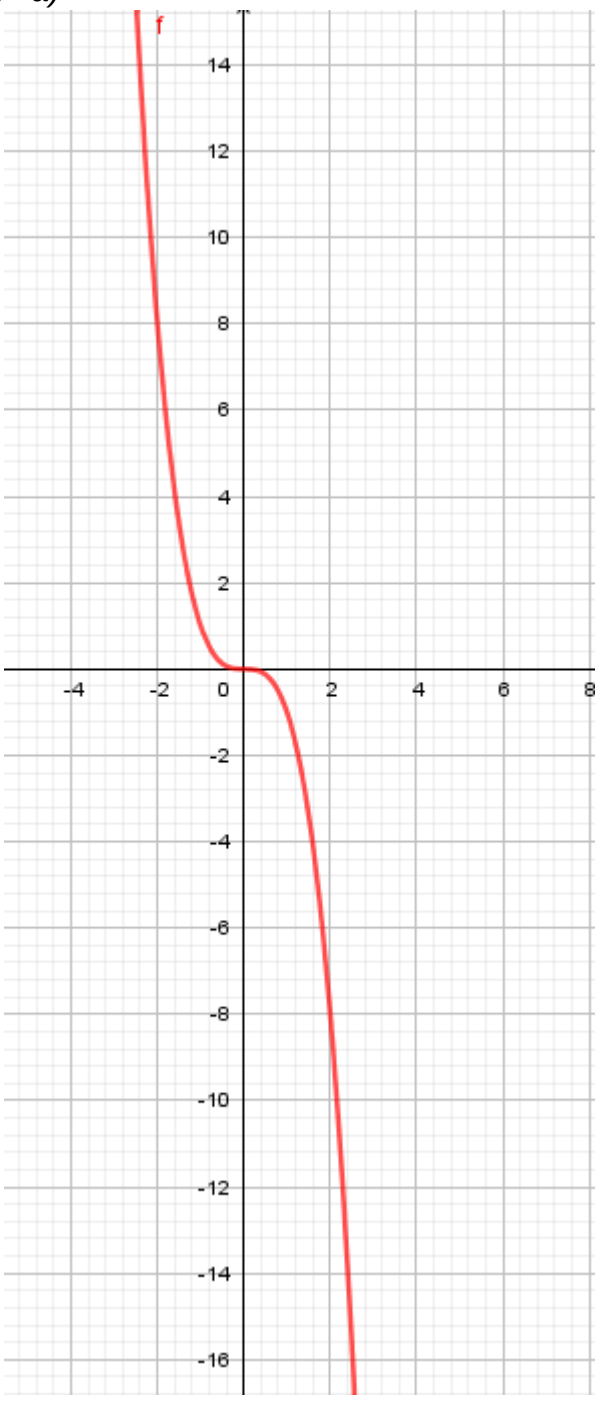
10) $f(4)=3$ $fg(x)=\frac{x+2}{3}$ $f^{-1}(x)=3x-5$
11) $f(x+2)=3x^3+16x^2+28x+20$
12) a =2, b = 3, $(x+3)^2 \geq 0$ *and so adding 2 means always positive.*

Drawing graphs and transformations of curves

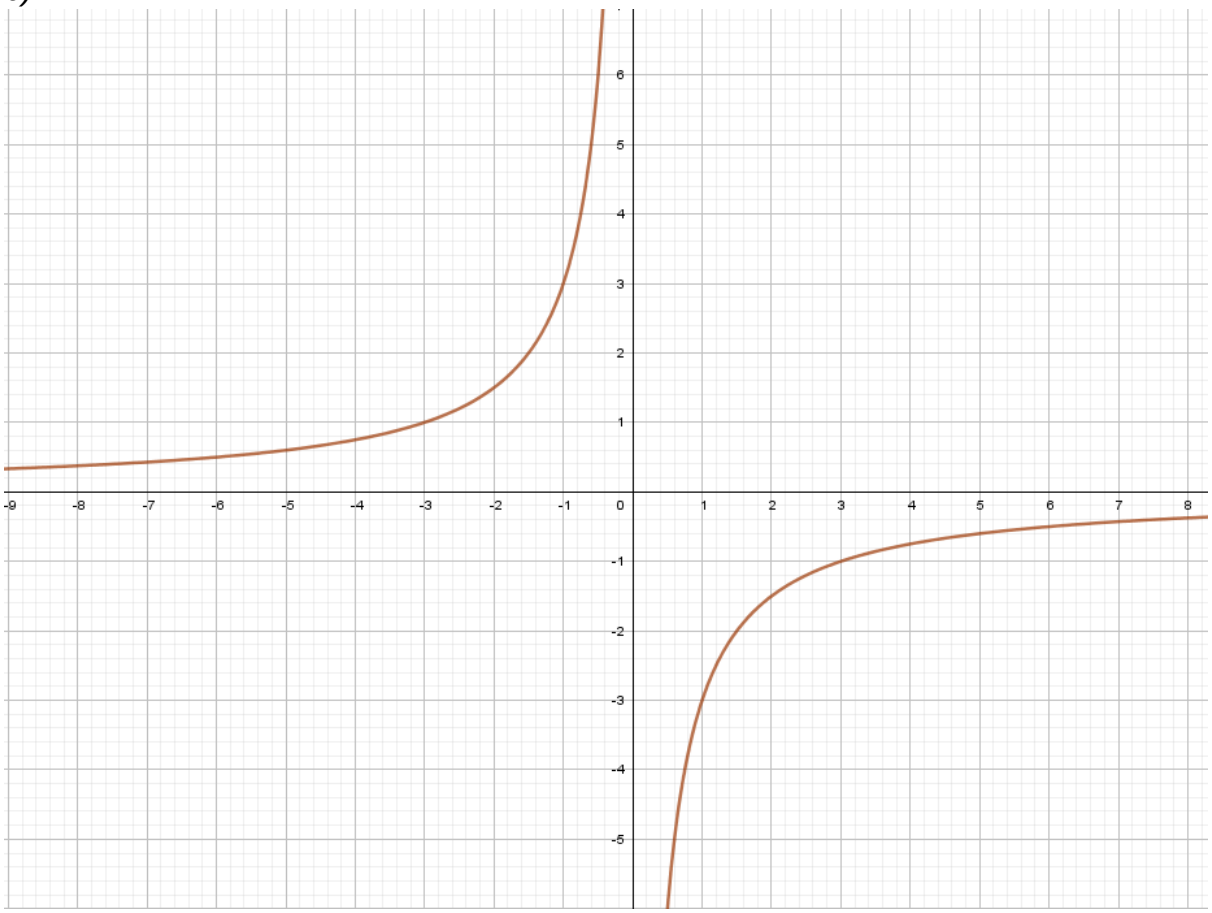
13) a) $y=2(x-\frac{5}{4})^2+\frac{71}{8}$ *so minimum point is* $(\frac{5}{4},\frac{71}{8})$

b) Curve does not intersect x-axis as minimum point is above it and the curve is always positive.

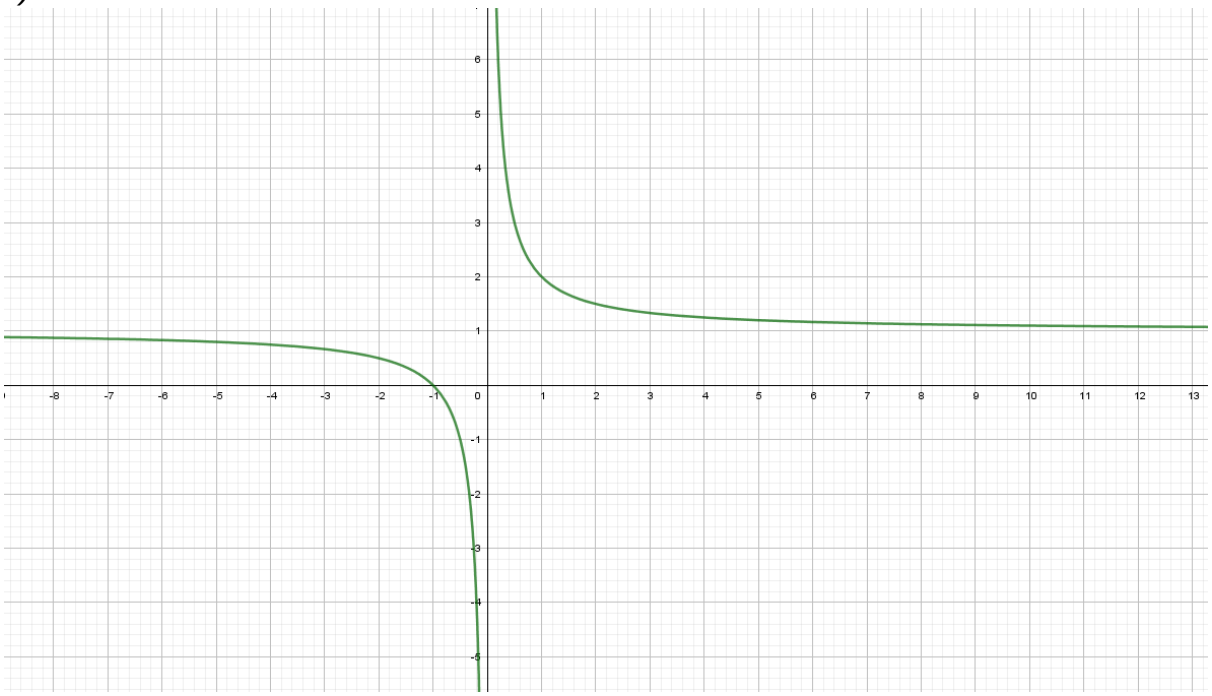
14) a)



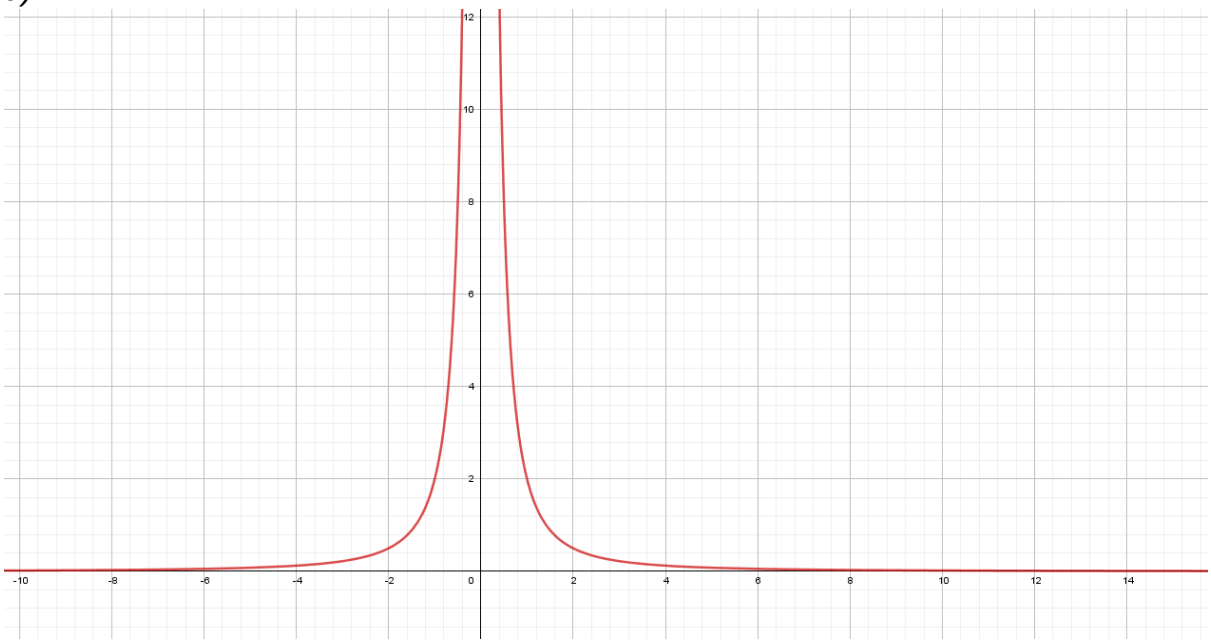
b)


-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8
6 5 4 3 2 1 -1 -2 -3 -4 -5

c)

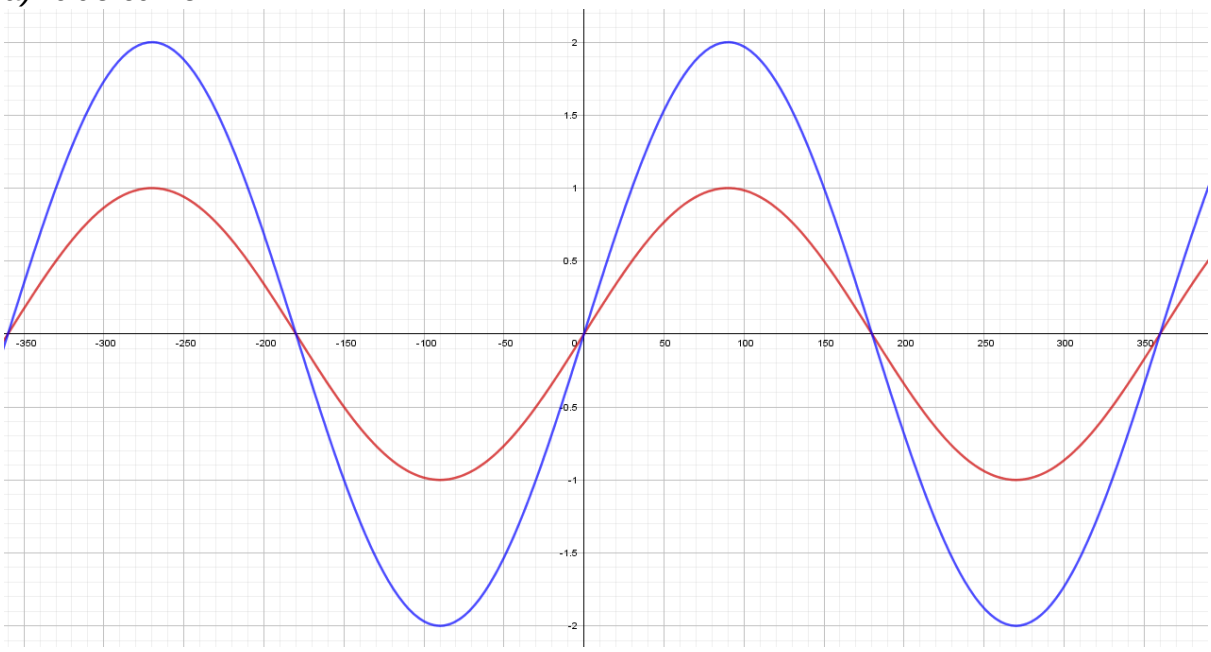

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13
6 5 4 3 2 1 -1 -2 -3 -4 -5

d)

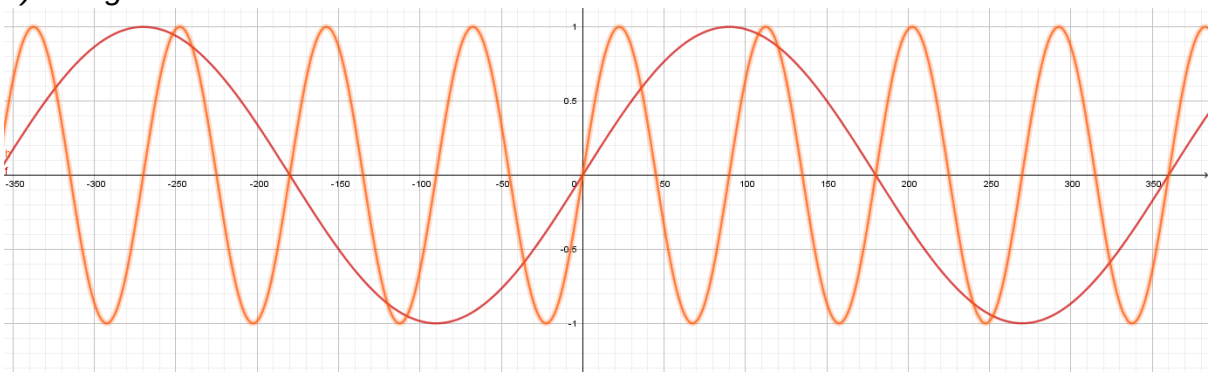


15)

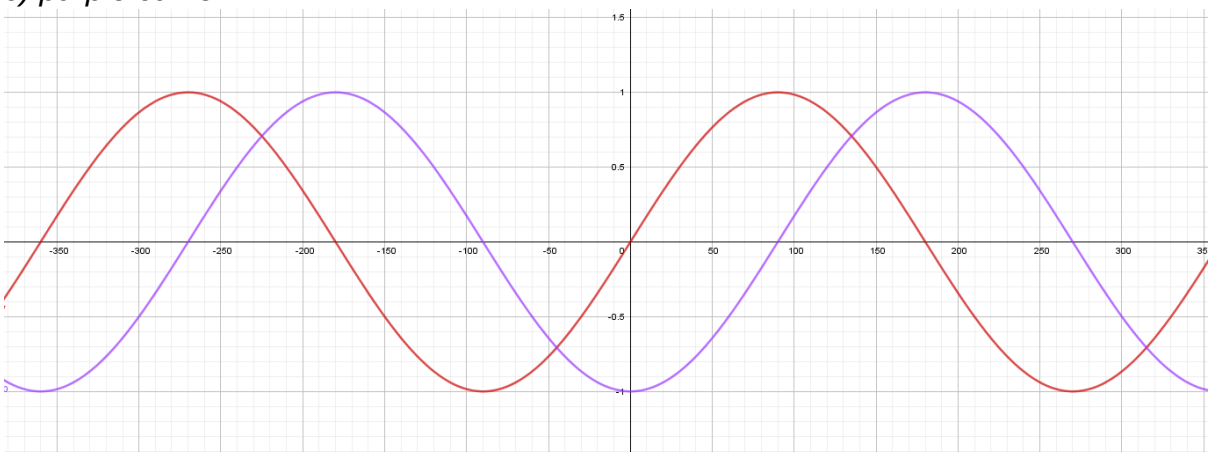
a) blue curve



b) orange curve



c) purple curve



16) a) (3,4) b) (5,1) c) (6,1)

3D Trigonometry and Pythagoras' Theorem

17) Workings in the table below

$(2n)^2 + n^2$
$(2n)^2 + n^2 + (n-1)^2 = (2n+1)^2$
$4n^2 + n^2 + n^2 - n - n + 1$ $= 4n^2 + 2n + 2n + 1$
$2n^2 - 6n = 0$
$2n(n-3) = 0$
$n = 3$

18) Workings in the table below

A triangle formed with A, B and the centre of the hemisphere with 2 sides of 10 cm and an angle of 120°
$(AB^2 =)\ 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 120$
$(AB =)$ [17.3, 17.321]
$(\cos AOB =)\ \frac{30^2 + 30^2 - \text{their } AB^2}{2 \times 30 \times 30}$
[33.557, 33.6]

Sequences

19) *Workings in the table below*

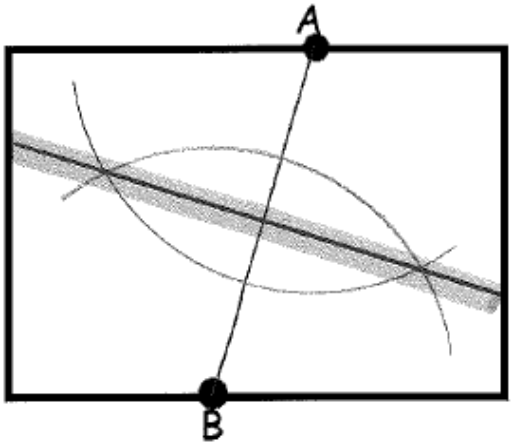
Method 1	*Method 2*
*n*th term of lengths is $n + 3$	*n*th term of 12 20 30
*n*th term of widths is $n + 2$	$= n^2 + 5n + 6$
Area is $(n + 3)(n + 2)$	
$n^2 + 3n + 2n + 6$ $= n^2 + 5n + 6$	

20) *Workings in the table below*

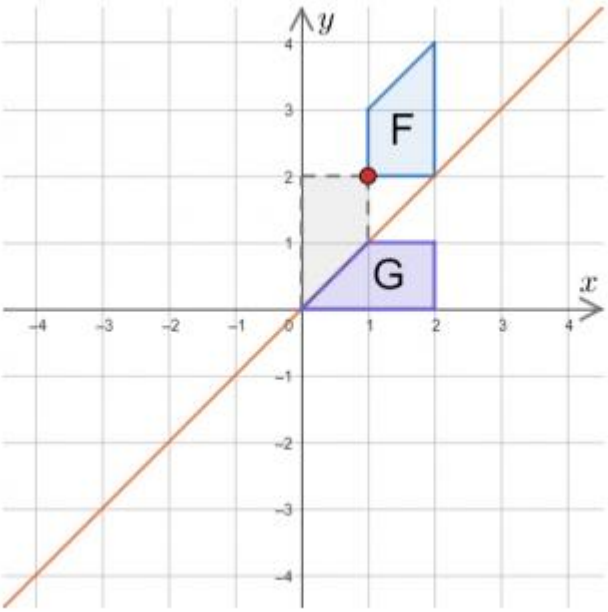
(a)	$a + 9b = 35$ $a + 15b = 59$ $6b = 24$ $b = 4$ $a = -1$
(b)	3 11 19 … … $8n - 5$

Transformations and Loci

21)

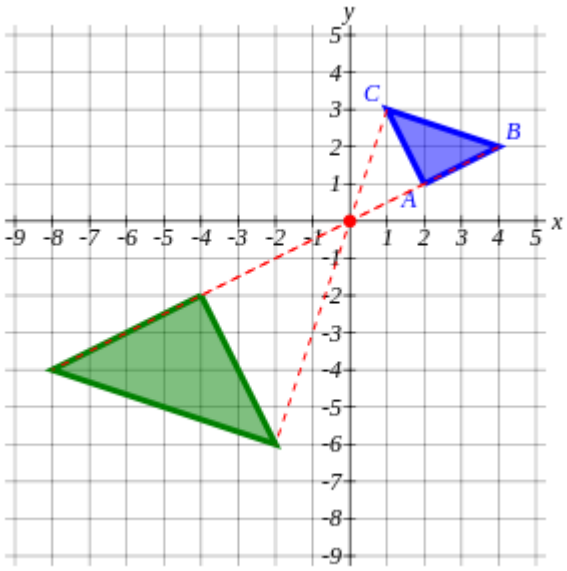


22)



Extension answer – There are no invariant points under the two transformations.

23) *Enlargement scale factor -2 centre (0,0)*



Vectors – Straight line vectors questions

24) a) $\overrightarrow{OP} = 2\boldsymbol{a} + 4\boldsymbol{c}$

b) $\overrightarrow{OM} = 3\boldsymbol{a} + 6\boldsymbol{c}$ $\overrightarrow{OP} = 2\boldsymbol{a} + 4\boldsymbol{c}$

$\overrightarrow{OM} = \frac{3}{2}\overrightarrow{OP}$ *therefore it is a straight line.*